IB MYP YEAR 5

YEAR 10 Mathematics

Assessment #3

VECTORS & MATRICES

Name: KWOK Chun Hei (10 Joy)

Teacher: **Ms. Li, Mr. So & Mr. Wong**

Date of task: **Friday, December 14, 2012** Time allowed: **95 mins**

Student's **Performance** in Different **Criterion**			
B	7	**D**	5

INSTRUCTIONS:

- Read the instructions for all questions carefully.
- Show all work, steps and proper units.
- Ask the teacher for scrap paper, but any work on the scrap paper will **NOT** be marked.
- Write in **PENCIL**.
- **NOT** allowed to use any **electronic devices**, such as translators.
- Allowed to use **GDC**.

ASSESSMENT:

- Read the criteria descriptors carefully before you start your work. This will give you a clear understanding of what is required and what a quality piece of work for this task must include. This way you give yourself the best chance of achieving the highest level in this task.
- This task assesses Criteria **B & D** considering ALL the questions.

Criterion B: INVESTIGATING PATTERNS **(ONLY APPLICABLE TO QUESTION IN PART 1)**

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-8)
1–2 Do Maths	The student performs appropriate calculations (in (a) to (c), in (g), to (i), and in (k) to (m)) in order to recognize simple patterns.	The student **applies**, **with some guidance**, mathematical problem-solving techniques to recognize **simple** patterns.	
3–4 General Rule	The student correctly solves (d) and (e) and suggests general rules in parts (f) and (n).	The student ● **selects and applies** mathematical problem-solving techniques to recognize patterns, and ● **suggests** relationships or general rules.	**Teacher's Final Grade**
5–6 Test it	The student describes relationships (in (j) and (n)) mathematically, and connects the various relationships. If a student has drawn conclusions consistent with their findings in part 2 (g), credit ***may*** be given here.	The student ● **selects and applies** mathematical problem-solving techniques to recognize patterns, ● **describes** them as relationships or general rules, and ● **draws conclusions** consistent with findings.	(0-8)
7–8 Prove it	The student is able to correctly prove mathematically all the relationships and rules seen in the task, and is successful in (j) and (n).	The student ● **selects and applies** mathematical problem-solving techniques to recognize patterns, ● **describes** them as relationships or general rules, ● **draws conclusions** consistent with findings, and ● **provides justifications or proofs.**	

Criterion D: REFLECTION **(only applicable to question in Part 2).**

Achievement level	Task Specific Rubric	IBO Published Descriptor	Student's self-evaluation
0	The student does not reach a standard described by any of the descriptors given below.	The student does not reach a standard described by any of the descriptors given below.	(0-6)
1–2	The student's answers to questions (a) and (b) describe the meaning of his/her findings in real life.	The student **attempts to explain** whether his or her results make sense in the context of the problem. The student **attempts to describe** the importance of his or her findings in connection to real life.	
3–4	The student's answers to questions (c) (i) to (iv) comment on how his / her findings make sense in real life situation.	The student **correctly but briefly explains** whether his or her results make sense in the context of the problem. The student **describes the importance** of his or her findings in connection to real life where appropriate. The student **attempts to justify** the degree of accuracy of his or her results where appropriate.	**Teacher's Final Grade** (0-6)
5–6	In question (d), the student is able to recognize and explain the applications seen in the vector operations. In question (e), the student has explained how this answer makes sense in the context of this problem.	The student **critically explains** whether his or her results make sense in the context of the problem. The student provides a **detailed explanation** of the importance of his or her findings in connection to real life. The student **justifies the degree of accuracy** of his or her results where appropriate. The student suggests improvements to his or her method where appropriate.	

A Special Matrix

Part 1

This section is assessed against criterion B only
You should spend 45-50 minutes on this section, and you are advised to make full use of your GDC.

Consider the following matrices:

$L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $M = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$ and $N = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

(a) Find the matrix L^2

$L^2 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 2\times2+1\times1 & 2\times1+1\times2 \\ 1\times2+2\times1 & 2\times1+2\times2 \end{pmatrix}$

$= \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$ ✓

(b) Find the matrix M^2

$M^2 = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 3\times3+2\times2 & 3\times2+2\times3 \\ 2\times3+3\times2 & 3\times2+3\times3 \end{pmatrix}$ ✓

$= \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$

(c) Find the matrix N^2

$N^2 = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 4\times4+3\times3 & 4\times3+3\times4 \\ 3\times4+4\times3 & 4\times3+4\times4 \end{pmatrix}$

$= \begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix}$ ✓

A student, Xavi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ then A^2 is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

(d) Find A^2

$A^2 = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix} \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

$= \begin{pmatrix} (a+1)^2 + a^2 & a(a+1) + a(a+1) \\ a(a+1) + a(a+1) & (a+1)^2 + a^2 \end{pmatrix}$

$= \begin{pmatrix} 2a^2 + 2a + 1 & 2a^2 + 2 \\ 2a^2 + 2a & 2a^2 + 2a + 1 \end{pmatrix}$

(e) Is Xavi right?

Yes.

Why?

(f) If your answer to **(e)** is "yes", is Xavi ***always*** right? Explain.
If your answer to **(e)** is "no", why is he wrong? Explain.

Xavi is always right because by comparing the elements of both matrices, it shows $b = 2a^2 + 2a$ so $b+1 = 2a^2 + 2a + 1$, but more importantly as matrix A is a diagonal matrix, meaning the values of diagonal elements are the same, so when matrix A has to times itself to derive A^2, it will still be a diagonal matrix.

(g) Find the matrix LM

$$LM = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times3+1\times2 & 2\times2+1\times3 \\ 1\times3+2\times2 & 2\times2+2\times3 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix}$$

(h) Find the matrix MN

$$MN = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3\times4+2\times3 & 3\times3+2\times4 \\ 2\times4+3\times3 & 2\times3+3\times4 \end{pmatrix}$$

$$= \begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix}$$

(i) Find the matrix LN

$$LN = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2\times4+1\times3 & 2\times3+1\times4 \\ 1\times4+2\times3 & 1\times3+2\times4 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix}$$

Another student, Messi, puts forward the following hypothesis:

If A is a matrix of the form $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$ and B is of the form $\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$

then the matrix AB is always of the form $\begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

(j) Prove (or disprove) Messi's hypothesis

$$AB = \begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}\begin{pmatrix} b+1 & b \\ b & b+1 \end{pmatrix}$$

$$= \begin{pmatrix} (a+1)(b+1)+ab & b(a+1)+a(b+1) \\ a(b+1)+b(a+1) & ab+(a+1)(b+1) \end{pmatrix}$$

$$= \begin{pmatrix} ab+a+b+1+ab & ab+b+ab+a \\ ab+a+ab+b & ab+ab+a+b+1 \end{pmatrix}$$

$$= \begin{pmatrix} a+b+2ab+1 & a+b+2ab \\ a+b+2ab & a+b+2ab+1 \end{pmatrix}$$

By comparing the two matrices,

$$\begin{pmatrix} a+b+2ab+1 & a+b+2ab \\ a+b+2ab & a+b+2ab+1 \end{pmatrix} \text{ and } \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$$

Considering $c = a+b+2ab$,

Messi's hypotheses is correct.

Now go back to the original matrices.

(k) Find L^3

$$L^3 = L^2 \cdot L$$
$$= \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 5\times2+4\times1 & 5\times1+4\times2 \\ 4\times2+5\times1 & 4\times1+5\times2 \end{pmatrix}$$
$$= \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$$

(l) Find L^4

$$L^4 = L^3 \cdot L$$
$$= \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 14\times2+13\times1 & 14\times1+13\times2 \\ 13\times2+14\times1 & 13\times1+14\times2 \end{pmatrix}$$
$$= \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$$

(m) Find L^5

$$L^5 = L^4 \cdot L$$
$$= \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$
$$= \begin{pmatrix} 41\times2+40\times1 & 41\times1+40\times2 \\ 40\times2+41\times1 & 40\times1+41\times2 \end{pmatrix}$$
$$= \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$$

(n) What do you notice about the form of the answers to (k), (l) and (m)? Try to use the various answers you have in order to generalize.

Given $L = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

Proven $L^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

$L^3 = \begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$

$L^4 = \begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$

$L^5 = \begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$

To derive L^2 from L,

$L^2 = \begin{pmatrix} 2+3^1 & 1+3^1 \\ 1+3^1 & 2+3^1 \end{pmatrix}$

To derive L^3 from L^2,

$L^3 = \begin{pmatrix} 5+3^2 & 4+3^2 \\ 4+3^2 & 5+3^2 \end{pmatrix}$

To derive L^4 from L^3,

$L^4 = \begin{pmatrix} 14+3^3 & 13+3^3 \\ 13+3^3 & 14+3^3 \end{pmatrix}$

To derive L^5 from L^4,

$L^5 = \begin{pmatrix} 41+3^4 & 40+3^4 \\ 40+3^4 & 41+3^4 \end{pmatrix}$

From the above derivations, it shows from L to L^2, 3^1 is added to each element; L^2 to L^3, 3^2 is added to each element; L^3 to L^4, 3^3 is added to each element; and L^4 to L^5, 3^4 is added to each element.

From the results, we are able to generalise this rule,

when $M = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$,

$M^2 = \begin{pmatrix} a+(a+b)^{2-1} & b+(a+b)^{2-1} \\ b+(a+b)^{2-1} & a+(a+b)^{2-1} \end{pmatrix}$

$M^3 = \begin{pmatrix} a+(a+b)^{2-1}+(a+b)^{3-1} & b+(a+b)^{2-1}+(a+b)^{3-1} \\ b+(a+b)^{2-1}+(a+b)^{3-1} & a+(a+b)^{2-1}+(a+b)^{3-1} \end{pmatrix}$

$\vdots$

$M^n = \begin{pmatrix} a+(a+b)^{n-x}\ldots & b+(a+b)^{n-x}\ldots \\ b+(a+b)^{n-x}\ldots & a+(a+b)^{n-x}\ldots \end{pmatrix}$

where x is a constant and where the value of elements added from $x=1$ until $x=n$.

Bad guy runs and cop chases... "*Catch me if you can!*"

Part 2

This section is assessed against criterion D only. You should spend 35-40 minutes on this section.

One day, a thief (T) stole a handbag from a lady (L) , the lady shouted for help, a cop (C) was nearby, he then ran after the thief.....

It is given that the position of T, C and L are (3, 14), (1, 8) and (k, -2k) respectively, where k is a constant.

(a) (i) Express $\vec{CT}$ in terms of **i** and **j**.

C(1,8), T(3,14)

$\vec{CT} = (3-1)\vec{i} + (14-8)\vec{j}$

$= 2\vec{i} + 6\vec{j}$

(ii) Express $\vec{TL}$ in terms of **i**, **j** and **k**.

T(3,14), L(k,-2k)

$\vec{TL} = (k-3)\vec{i} + (-2k-14)\vec{j}$

$= (k-3)\vec{i} - (2k+14)\vec{j}$

(b) If the value of k is -1, describe the meaning of this situation in real life. Show your work briefly.

When $k=-1$, it signifies the lady, thief and cop are on the same line, or represented as being collinear if their points are on a coordinate plane.

$m_{TC} = \frac{14-8}{3-1} = \frac{6}{2} = 3$

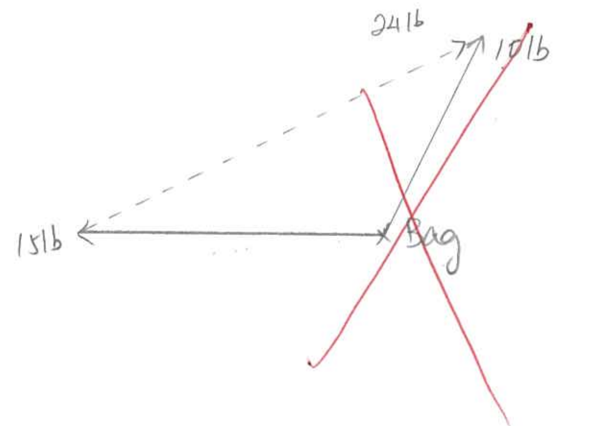
$m_{TL} = \frac{14-2}{3-(-1)} = \frac{12}{4} = 3$

$\therefore m_{TC} = m_{TL}$

$\therefore$ Points T, C and L are collinear.

A few minutes later, T and C met… they started fighting for the handbag. Two forces with magnitudes of 15 lb and 10 lb were applied to the bag. The magnitude of the resultant was 24 lb.

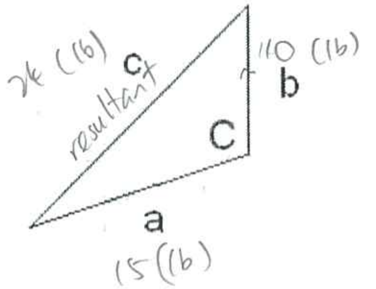
(c) (i) Sketch a diagram to show each force in component form and the resultant force.



C

(ii) Find the measurement of the angle between the resultant vector and the vector of the 10 lb force to the nearest degree.

Hint: use cosine rule **$c^2=a^2+b^2-2(a)(b)(\cos C)$**



By cosine rule,

$(24)^2 = 15^2 + 10^2 - 2(15)(10)(\cos C)$

$576 = 225 + 100 - 300\cos C$

$300\cos C = -251$

$\cos C = \left(\frac{-251}{300}\right)$

$C = \cos^{-1}\left(\frac{-251}{300}\right)$

$= 147^\circ$ (corr. to 3 sig fig.)

(iii) According to the above situation, the two forces from different directions are applied to the handbag, can the magnitude of the resultant be larger than the sum of the magnitudes of the forces? Justify your answer.

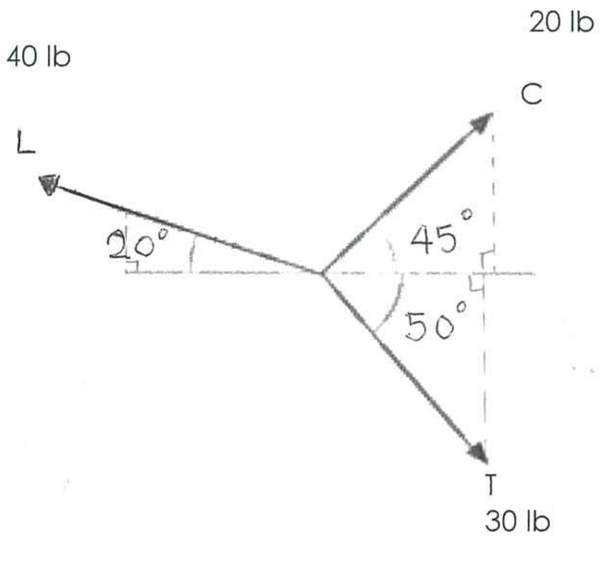
No, the magnitude of the resultant force cannot be larger than the sum of the magnitudes of the forces because 'resultant' means there is a leading force which one side is able to succeed in the given condition. having the resultant force larger than the hand bag will only signify there is an external force acting upon the circumstances. With calculations, as the resultant force is determined by using Pyth. Theorem when the forces are given or proven, which the value of hypothenuse side cannot be greater than the sum of the opposite and adjacent side.

(iv) What if the forces were from the same direction? How would it affect the magnitude of the resultant force? Does it make sense in real life? Briefly justify your answer.

The magnitude of the resultant force cannot be justified because the forces are on the same line of a coordinate plane, which a triangle cannot be formed to show direction of vector.

This does not make sense in real life because when a thief and a cop is fighting for a handbag, they would probably be at different directions of the coordinate plane, but not at the same point, also, as a nature, people can't be fighting for a thing at the same direction, with one forward and backward, there should be some difference in term of direction between them.

Finally, L caught up with T and C, she joined the fight as well. The diagram shows the 3 forces.



(d) The three forces shown in the diagram act at a point. Find the magnitude of their resultant and draw a diagram to show its directions. Who would have greater chance to get the handbag?

Force of T: $\begin{pmatrix} 30\cos 50^\circ \\ -30\sin 50^\circ \end{pmatrix}$

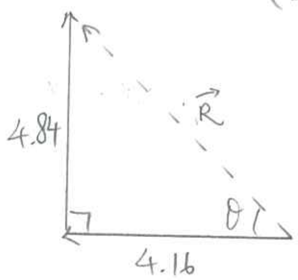
Force of C: $\begin{pmatrix} 20\cos 45^\circ \\ 20\sin 45^\circ \end{pmatrix}$

Force of L: $\begin{pmatrix} -40\cos 20^\circ \\ 40\sin 20^\circ \end{pmatrix}$

Resultant Force: $\begin{pmatrix} 30\cos 50^\circ \\ -30\sin 50^\circ \end{pmatrix} + \begin{pmatrix} 20\cos 45^\circ \\ 20\sin 45^\circ \end{pmatrix} + \begin{pmatrix} -40\cos 20^\circ \\ 40\sin 20^\circ \end{pmatrix}$

$= \begin{pmatrix} 30\cos 50^\circ + 20\cos 45^\circ - 40\cos 20^\circ \\ -30\sin 50^\circ + 20\sin 45^\circ + 40\sin 20^\circ \end{pmatrix}$

$= \begin{pmatrix} -4.16 \\ 4.84 \end{pmatrix}$ (corr. to 3 sig. fig.)



By Pyth. Theorem,

$\vec{R} = \sqrt{4.16^2 + 4.84^2}$

$= 6.38$ (lb)

By tangent,

$\tan\theta = \frac{4.84}{4.16}$

$\theta = \tan^{-1}\left(\frac{4.84}{4.16}\right)$

$= 49.3$ (corr. to 3 sig fig.)

$\therefore$ Direction is N30.7°W.

$\therefore$ The magnitude of their resultant force is 6.38 lb, and the direction is N30.7°W.

$\therefore$ The lady (L) would have a greater chance to get the handbag.

(e) Explain how this answer makes sense in the context of this problem.

The answer makes sense in the context of this problem because that the lady gets the handbag

the thief and cop has already fight for the bag for quite a while before the lady. Within that period of time, the thief and cop have probably used most of their energy, so when the lady arrives, she is actually the most powerful person amongst the three of them, hence it is possible for her to get the handbag.

End of Assessment

75 43 . 45 . 43 .